What has been achieved through the application of statistics to numismatics ?

In the Commonwealth of learning and research, each of us is at liberty to be sovereign in thought and judgment : it is this individual independence that is safeguarded by the convention of academic freedom. Each of us is also at liberty to set aside his self-interest and to seek to be helpful to his fellows in other ways than by his own pursuit of excellence. Such disinterested help is particularly precious in collaboration between disciplines – as for example between statistics and numismatics on this occasion. But when we speak different languages (on the one hand the language of science and on the other that of letters) problems of communication can arise. Research effort may be misdirected, through a failure to identify the problems exactly. The door is then opened to contributions so scrappy as to verge on triviality, and promise may pass for achievement.

The role of the statistician is, evidently, that of an expert whom numismatists consult, in order to obtain advice and help with questions which are beyond their competence. The personal initiative of the statistician may go further, but if it does, he will have become to some extent a numismatist himself, and will be playing two roles, for the questions on which his expert advice is sought are numismatic questions. It is no longer prudent for the statistician to approach these inter-disciplinary problems without acquainting himself with what has already been achieved : there is a growing body of published work to guide him. Many of the gains have been made here in Paris, and we are all indebted to the *Société française de Numismatique*, to their *Journée d'études* in 1974[1] (which was a culmination of several years' interest), and in particular to Julien Guey, for their efforts to consolidate and promote what is known. This accumulated wisdom should be familiar ground to the statistician. The context of the numismatist's particular questions, however, will almost by definition be unfamiliar,

1. The proceedings were published in a special number of *BSFN*, for July 1974.

and there is a temptation not to spend time and effort in mastering it, but to solve generalized or academic problems instead. It may for that reason be useful to take a hard look at what has actually been achieved, in terms of lasting contributions to the body of numismatic knowledge. When our researches are eventually taken into account in standard works on coinage, we may tell ourselves that they have been of some help.

Among the questions that numismatists ask, we may distinguish two broad categories. First, there are questions relating to the work of the mints. They concern, for example, the weight and alloy of the coins, and the quantities in which they were struck. If these questions could be fully and exactly answered, we should be able to compile statistical summaries resembling those which the officials of a modern mint present to the government: «I have the honour to report that during the year –, so many million coins were struck, at such and such weight and fineness.» Information of this kind about ancient or medieval coins can lead us towards an understanding of the intentions of the issuing authorities. Secondly, there are questions which seek to explore what happened to the coins after they were issued: loss of weight by wear; changes suffered by the alloy through corrosion, etc.; and the wasting away of the stock of currency by export, withdrawal, reminting, hoarding, and so on. Thus we have three topics twice over, and we shall in a few moments look at each of them in turn, beginning with the production of the coins and their weight-standards. Any detailed consideration of the alloys used in coinage is in principle excluded on this occasion.

For generations, numismatists have grown up in a tradition of antiquarianism, and their natural instinct has been to think of the coin in their collection as an individual object, of which the essential testimony was contained in its portrait, its reverse design, its inscriptions. Coins have been regarded, in other words, rather as though they were medals. The preliminary achievement of statistics as applied to numismatics has been to give greater prominence to the idea that coins were produced in enormous quantities (even though they may have had a low survival-rate), and for mundane purposes. Today, the pictures on your bank-notes may be beautiful, but they are purely decorative, and are of no relevance to the purchasing-power of the currency. Numismatists will always find it painful to admit that something similar may be true of their coins. Nevertheless, the possibility of undertaking more quantitative studies has encouraged an extension of their field of interest, towards monetary circulation and research into the role of coins as money. Let us keep this development in perspective: the volumes of the *Revue Numismatique* for 1976 and 1977, which are full of good things, each contain one article with a statistical approach, against a dozen which still rely entirely on traditional modes of numismatic enquiry.

The realities of monetary history are not particularly romantic. Long before the Industrial Revolution, coins were mass-produced objects. If they had an intrinsic value, careful quality control was essential; and it was necessary for their public acceptability that they should all look identical.

Their weights were for obvious reasons, then, prescribed by the authorities. From the fourteenth century onwards, or occasionally even earlier, documents have survived which tell us how the weight-standard was determined – and also how the coins were sampled, to ensure that they complied with the standard. Often it was laid down that so many coins were to be made from each pound or other unit-weight of metal, and usually the legal tolerances were also defined[2]. The Trial of the Pyx (in England) provided a means of control[3]. But for most medieval currencies, and for virtually the entire Byzantine coinage, and the entire coinage of the ancient world, the mint-documents have not survived, and our attempts to recover the terms in which the weights were regulated depend not so much on modern weighings of surviving coins nor on their statistical treatment – although those procedures are obviously necessary – as on assumptions about administrative continuity and conservatism, and on comparisons between the coinage systems at different dates or in neighbouring countries. These assumptions and comparisons need to be weighed just as carefully, – even though it should be possible to state the approximate average weight of different coin denominations in modern terms without venturing into any delicate speculations about the original terminology.

It is not only the average weight, but also (and perhaps even more) the variations on either side of the average which merit statistical study. The blanks on which the coins were struck were cut by skilled workmen using their best judgment, but naturally some were made a little too light, and some a little too heavy. It was often the practice at that stage to test the blanks and to reject those which fell outside the legal tolerance, or which were very inaccurate in weight. This was done more rigorously with precious metals than it was with copper and bronze, since heavy flans would otherwise promptly be picked out by the users and melted down or exported. The resultant distribution of weights of the newly-minted coins may approximate, therefore, to a modified Gaussian curve, truncated perhaps at the lower but more particularly at the upper extreme. Empirically we often find a negative asymmetry with a tail of very low values, and it may be that this is a characteristic pattern at the moment when the coins left the mint.

2. For French examples, see LAFAURIE, 1951, and the bibliography cited there; the fullest information, however, and a wonderful testing-ground for theories of every kind, is available for the English sterlings of the Edwardian period. For critical surveys and bibliography, see N.J. MAYHEW (ed.), *Edwardian Monetary Affairs (1279-1344), A Symposium held in Oxford, August 1976 (British Archaeological Reports*, XXXVI), 1977.
3. STIGLER, 1977.

The statistician's approach has necessarily been to begin by adopting a model to which the data seem to approximate, but which is a random model *(un modèle aléatoire)*, and the preliminary assumption has been that, through the processes of manufacture, the coins originally acquired, for instance, a Gaussian distribution. One can then go on to quantify the ways in which the empirical data deviated from this expectation. We can, however, rarely be confident that the coins available to us correspond in their weight-distribution with freshly-minted pieces. Attempts to recover the parameters describing the original distribution of weights have nevertheless sometimes been the fertile ground and starting-point for numismatic research. Petersson's thorough survey of multiple weight-standards within individual Anglo-Saxon coin-types is an example[4].

It is always as well to look at the mode as well as the mean of a sample of coins, even though the mode is of no further use as a base for extended statistical calculations. If one is seeking to be cautious in fitting the data to a Gaussian distribution, the mode may arguably be a useful indicator of the standard to which the flans were cut. When the sample is small there may be practical difficulties in defining the mode[5]. A formula for calculating it has been discussed by Suchodolski[6].

So much for weight-standards and the original weights of the coins. Similar considerations apply to the alloy of ancient and medieval coinages: the survival from about the fourteenth century of mint-prescriptions; the character of our speculation about the terms of earlier alloy-standards; the possibility of at least approximate measurements in modern chemical terms; and the difficulty of interpreting observed parameters[7].

The third category of statistical information about mint-output (using the word statistical in the economist's sense of a digest, not quite the same as the mathematician's) concerns the volume of coinage struck. It is here that mathematical or predictive statistics makes a contribution to numismatics and monetary history which is novel, obtainable in no other way, and of real importance to historians[8]. The end of the Ancient World, and the

4. Petersson, 1969.
5. A practical procedure for studying the mode is offered in Metcalf, 1971, at p. 203-6 and figs. 10 and 11.
6. Suchodolski, 1965, p. 49, note 14a. The article by Zabiński described there as being « sous presse » seems never to have been published.
7. For this subject, see E.T. Hall and D.M. Metcalf (ed.), *Methods of Chemical and Metallurgical Investigation of Ancient Coinage (Royal Numismatic Society, Special Publication*, VIII), 1972. For a sensible statistical approach to the parameters of fineness, see D.R. Walker, *The Metrology of the Roman Silver Coinage*, 3 vols. (*British Archaeological Reports, Suppl. Ser.*, V, XXII, and XL), 1976-8.
8. Until the 1950's, it was possible for exaggeratedly low estimates of die-output, e.g. 1 000 coins per anvil die, and 500 to 800 per reverse die to gain acceptance. See Brunetti, 1963a, and Sellwood, 1963. In the eighteenth century, it was even seriously held that only one coin was struck from each die.

Dark Ages of early medieval Europe, have been perceived by historians as a time when the Roman monetary economy collapsed and vanished, and when the whole of northern Europe sank into a regime of local self-sufficiency and barter. This persisted, so it was said, until the Crusades gave a new stimulus to long-distance trade and to a money economy. Once again, documentary records of mint-output survive only patchily from about the fourteenth century onwards[9]. For all earlier periods, statistical estimation based on random sampling procedures allows us to calculate the probable numbers of dies that were used to strike a coinage. When it was first exploited, this appeared to be a straightforward application of the statistical theory of probability and sampling to be found in any textbook, and various scholars derived methods independently: Brunetti was a pioneer[10]. Brown[11] and Lyon[12] followed. A useful critical exposition of Brown's formula has since been given by Guilbaud[13]. But the coin evidence turned out to be less than straightforward. Lyon was soon able to show that a theory was needed which took into account the fact that there was wide variation in the output of individual dies[14]. The point was reinforced by Grierson and by Brown[15], and Lyon later published, in a monograph about the mint of Lincoln, a new method of estimation, borrowed from the work of the biologist Good, which yields a measure of the total original output rather than the original total number of dies implied by the sample[16]. The « Lincoln » formula has the two great merits of mathematical simplicity and of taking account of variable die output. Both these points should commend it to numismatists, for there is ample evidence that, within an issue, the output of individual dies normally varied in a way that can be plotted as an approximately exponential curve. Indeed, one may go so far as to suggest that formulas which assume equal die-ouput should not be used except for good reason. One hopes that this characteristic feature of the numismatic evidence, namely very unequal die-output, will be firmly grasped by

9. The survival of mint-records in England is exceptionally full, and the evidence is of general interest to students of other series. The records are summarized, with references, in CRAIG, 1953. Various additions and corrections since then will be found in the following: C.E. BLUNT and J.D. BRAND, *Mint output of Henry III*, in *BNJ*, XXXIX, 1970, p. 61-6; M. MATE, *Monetary policies in England, 1272-1307*, in *BNJ*, XLI, 1972, p. 34-79; C.E. CHALLIS, *The Tudor Coinage*, 1978, p. 305-8; P. WOODHEAD, *Calais and its mint. Part two, Coinage in the Low Countries (880-1500)*, 1978, p. 185-202. For French coinage, see H.A. MISKIMIN, *Money, Prices and Foreign Exchange in Fourteenth-century France*, 1963, Appendix D.
10. BRUNETTI, 1950/1, etc.
11. BROWN, 1955/7.
12. LYON, 1965.
13. GUILBAUD, 1974.
14. LYON, 1966.
15. GRIERSON, 1968; BROWN, 1969.
16. LYON, 1970.

statisticians – for elaborations of the earlier statistical theory, and new methods derived from first principles, are still being published[17]. When die-output is demonstrably very variable, their value is in doubt.

The annual average output of medieval dies is known in a wealth of detail from fourteenth-century documentation, the figures published by Mrs. Mate being particularly instructive[18]. They provide the best indication we have, albeit not a very satisfactory one, of average die-output in the ancient world. Annual averages of 15,000 to 20,000 coins from each reverse or upper die, and twice as many from an obverse or lower die, are commonplace for medieval silver. Dies for medieval gold were less heavily used – apparently much below their technical capacity – in the interests, one may suppose, of better quality of workmanship[19]. For earlier centuries, and for coins in higher relief or larger size, numbers in the range of 5,000 to 15,000 may suggest themselves as round estimates of the technical capacity of an upper die. It must be stressed that these figures are purely nominal, that as an estimate of actual annual average die-output they could be in error by a factor of two, depending for example on whether the coins were struck hot or cold, and that there is very little we can do to remedy the deficiency. Thus it remains to some extent a matter of disagreement how plentiful coinage was in the early middle ages. But where the evidence is that fifty or a hundred dies *of the same design* were used, it is hard to see why so many should be made unless they were needed, i.e. unless most of them were fully used. For the Ancient World in general we move several centuries further back into the realms of untested hypothesis. Hence the considerable interest and importance of Mattingly's historical arguments, which indicate that the actual average output of an upper die for denarii in Roman Republican times is to be sought in the region of 12,000 to 15,000[20].

For the historian's purpose of judging the volume of the currency, statistical refinement in estimating the numbers of dies used may well be superfluous, except that one can usefully make comparisons between similar groups of coins assuming that average die-output was approximately constant, as for example in successive periods. Accuracy within narrow limits is in any case not needed to argue the proposition which is of

17. MORA, 1977; ESTY, 1978.
18. A critical appraisal of the documentary records about dies (STEWART, 1963, 1964) is supplemented and placed beyond dispute by MATE, 1969.
19. See, for example, W.J.W. POTTER, *The silver coinage of Edward III from 1351*, in *NC*, 1960, p. 137-81, at p. 174; and D.M. METCALF, *Coins of Henry VII (Sylloge of Coins of the British Isles)*, 1976, p. xxvi.
20. SELLWOOD, 1962, suggests 8 000 per upper die (hot striking) or 4 000 (cold). The figures are really only a guess, and may have been influenced by the much too low estimates that had been proposed previously. They are now in need of revision in light of Mattingly's comments (see below).

primary interest for the history of the early middle ages, namely that coins were often struck in quantities measured in millions, in periods during which historians have supposed that a money economy was moribund[21]. The case is strengthened, and our understanding of it is improved, more by increasing the number of separate samples, of different issues of coins, than by refinement of the statistical techniques applied to a few existing samples.

Statistical estimation has provided working figures for thirty or forty series of medieval coins, among them Byzantine of the 6th[22] and the 8th to 14th centuries[23], Carolingian[24], Anglo-Saxon from the 7th to the 11th centuries[25], English[26], Scottish[27], Low Countries[28], German[29], Crusader, Armenian[30], and Balkan[31]. New soundings are being added year by year. They have already accumulated to the point where they are beginning to establish the outlines of a quantitative view for European monetary history and its economic interpretation. Without the application of statistics to numismatics, this information would have remained totally inaccessible and unknown. It represents a distinctive contribution to medieval history, which is perhaps of greater general interest than any other result of the use of statistics in numismatics.

In ancient numismatics, particularly Greek and Roman, the exploration

21. A rash attempt to advance this view for eighth-century England (METCALF, 1963a) provoked a hostile reaction (GRIERSON, 1963a) and led to a drawn-out dispute (METCALF, 1963b; GRIERSON, 1963b, METCALF, 1964).
22. D.M. METCALF, *The Copper Coinage of Thessalonica under Justinian I*, Vienna, 1976, p. 8 and 29-31, referring also to a comparable estimate for the reign of Anastasius I.
23. ID., *Coinage in South-eastern Europe, 820-1396 (Royal Numismatic Society, Special Publication*, XI), 1979, at p. 10-12, etc.
24. METCALF, 1967. A fuller study of the early Carolingian series is in progress. For die-studies of the later reigns, see H. ENNO VAN GELDER, *Le trésor carolingien d'Ide*, in *RN*[6], VII, 1965, p. 241-61, and J. LAFAURIE, *Deux trésors monétaires carolingiens : Saumeray (Eure-et-Loire), Rennes (Ille-et-Vilaine), ibid.*, p. 262-305.
25. METCALF, 1965, 1967; LYON, 1970; METCALF, 1978.
26. ID., 1977.
27. STEWART, 1977.
28. P. NASTER, *Les deniers d'Arnot émis à Alost*, in *RBN*, XCVIII, 1952, p. 41-55.
29. W. HAHN, *Beiträge zu einem Stempelcorpus der bayerischen Münzen des 10. und 11. Jahrhunderts*, in *Jahrbuch für Numismatik und Geldgeschichte*, XXVII, 1977, p. 79-92; ID., *Herzog Heinrich II. von Bayern und die Anfänge der böhmischen Münzprägung*, in *WN*, XXI, 1977, p. 162-7; ID., *Die Regensburger Münzprägung unter Herzog Otto von Schwaben und Bayern (976-982)*, in *Beiträge zur süddeutschen Münzgeschichte. Festschrift zum 75-jährigen Bestehen des Württembergischen Vereins für Münzkunde*, Stuttgart, 1976, p. 58-72.
30. D.M. METCALF, *Coinage of the Latin Kingdom of Jerusalem in the name of Baudouin*, in *NC*, CXXXVIII, 1978, p. 71-84; C. SABINE, *The Turris Davit coinage, ibid.*, p. 85-92, at p. 91. P.Z. BEDOUKIAN, *Two hoards of Hetoum-Zabel trams*, in *ANSMN*, VIII, 1958, p. 145-80; ID., *A hoard of bilingual trams of Hetoum I of Cilician Armenia, ibid.*, XXIII, 1978, p. 149-60; ID., *The double tram series of Levon I of Cilician Armenia*, in *NC*, CXXXVI, 1976, p. 98-108.
31. D.M. METCALF, *Coinage in South-eastern Europe, 820-1396*, 1979. Good information exists for the mint of Dubrovnik.

of quantities has not proceeded as far, in spite of the availability of numerous careful die-studies. Brunetti's calculations of the 1950's and 1960's are discredited, but his numismatic data are available for re-use, indeed are crying out to be exploited. There has been some very useful debate about the volume of issues in the Roman Republican period[32], and scattered comment on later centuries, but no attempt as yet to draw perspectives for the ancient world as a whole. Whereas there is an extremely rich literature surviving from the classical world, our quantitative grasp of its economic history is for the most part far more vague and uncertain than for the Middle Ages. If eventually comparisons can be made across the gulf of the Dark Ages, they may help to define the ways in which coinage was used in ancient times. Equally, comparisons of the volume of coinage remain to be made between the classical world and the Celtic cultures on its northern fringes – and similarly with the Indo-Greeks and Parthians.

We distinguished two broad categories of information, of which the first related to the work of the mint, namely the weights and alloys of the coins, and the quantities struck. To this may be added some supplementary data on the techniques of production: die-alignment, for example, is susceptible of statistical treatment, as M^me^ Carcassonne has shown[34].

The other category of information concerns what happened to the coins after they left the mints, and here there are no simple or neat answers, because the fate of individual coins varied. The application of statistical procedures once again falls into three main compartments, namely weight-variation, alloy-changes, and quantities of coins in circulation. The areas of disagreement or uncertainty are smaller, but the questions are more delicate, and it is desirable to define them as clearly as possible, so that the statistician may deploy his skills in examining the real problems, and not some partial approximation to them.

Weight-variation is a case in point. Metrology is of course a subject in its own right, which exceeds the scope of our present discussion. The application of statistics to it has demonstrated the usefulness to numismatics of some powerful techniques, for example the analysis of variance, and the application of the χ^2 test[35]. The use of standard statistical procedures has generated an extensive bibliography, and very small differences have been keenly debated. The statistical niceties have preoccupied the participants, and less attention has been given than it perhaps deserves to defining the

32. MATTINGLY, 1977, argues for an average output per reverse die of the order of 12 000-15 000, rather than the c. 25 000 proposed by M.H. Crawford, whose *Roman Republican Coinage* (2 vols, 1974) is a major work of monetary history.
33. See, for example, C.H.V. SUTHERLAND, *The Cistophori of Augustus (Royal Numismatic Society, Special Publication,* V), 1970, p. 107f.
34. MILLIAU, 1960; GUEY and CARCASSONNE, 1970; CARCASSONNE, 1974b.
35. CARCASSONNE and GUEY, 1972; AUBIN and CARCASSONNE, 1977.

numismatic and historical objectives. Grierson's clear-sighted comments[36] are essential background reading.

Gold coins, which are not subject to corrosion, and which receive far less wear than silver or copper coins, are by general agreement the best evidence from which to proceed. French gold coins in the nineteenth century lost about 1 mg a year if they were in moderately active circulation, and about 2 mg in very active circulation. This was irrespective of the denomination, so that a 20-franc piece would lose 0,15 % of its weight per decade, while a 5-franc piece lost as much as 1,2 % per decade, or 8 times as much in relation to its weight. Silver 2-franc and 1-franc coins lost about 10 mg a year, thus 1 % or 2 % per decade respectively. In England in 1798 similar rates of wear, possibly exacerbated by culling, had resulted in the shillings in circulation being 24 % deficient in weight, and the sixpences 38 % deficient. Again, Grierson's comments on the available figures are the best guide to their numismatic usefulness[37].

For the estimation of weight-standards, the important argument to consider is that in antiquity and the middle ages, gold coins which were hoarded more than they were circulated (and which sometimes circulated in sealed purses or bags) are likely to have suffered very little loss by wear – even less, perhaps, than the 0,15 % per decade of the modern French 20-franc pieces, for they were sometimes hoarded in superb condition. Such coins are evidence of the highest quality. The exact weight of the Roman and Byzantine pound has been vigorously discussed, with opinion swaying between about 322 and about 327 g. Hyper-accuracy is specious : Guey has recently given us an elegant demonstration of the statistical impossibility of deriving an answer to the nearest centigramme from the available evidence. In practice a figure correct to the nearest gramme is extraordinarily difficult to achieve.

When we say « *the* Roman and Byzantine pound », we should not lose sight of the probability (which Hultsch discussed[38]) of minor ponderal changes occurring from time to time. It is very far from true that the weight of the Imperial gold coinage was immutable for many centuries – as Walker's recent survey of Roman numismatic metrology[39] reminds us. Nor is there unassailable documentary evidence that the mints shared exactly similar weight-standards, or applied them with equal care, or that they

36. Grierson, 1963c, 1964.
37. Nanteuil, 1927-8 ; Chaston, 1974 ; Grierson, 1963c.
38. F. Hultsch, *Griechische und römische Metrologie*[2], Berlin, 1882, p. 158-61 : « Gegen Ende des vierten Jahrhunderts scheint freilich eine kleine Verringerung [diminution] des Pfundes eingetreten zu sein. Dies beweist sowohl der etwas sinkende Fuss der Solidi... » Ponderal reforms, including the authorization of new standard weights for the mints, in the early modern period are perhaps instructive here.
39. Walker, *op. cit.* (note 7).

always produced exactly 72 coins from a pound of gold[40]. Indeed, M^me^ Morrisson has presented valuable empirical evidence from the seventh century which indicates that the authorities were capable of making a deliberate reduction in the standard of the order of 1 or 2 per cent[41]. Note that this is ten times the loss of weight per decade found in 20-franc pieces: the idea that weight-standards were adjusted to match the loss of weight in a currency by wear is not in accordance with the available measurements, at least for gold. Ponderal systems display an extremely high degree of inertia in their concepts and base-points, but at the same time they suffer from small creeping changes, periodic reforms, and regional mutations. A good idea of these last can be obtained from Miskimin's study of medieval systems[42], which as usual are better documented. On the statistical front, increasingly sophisticated attempts have been made to proceed from the known to the unknown, that is, from observed weight-distributions to the weight-distribution of the coinage when it left the mint[43]. Let us remind ourselves that the difference is the resultant of numerous tendencies, some of them conflicting, such as loss of weight by wear, change of weight by corrosion and leaching, loss of weight by clipping or filing, the assiduous sifting of the currency by numerous individuals in search of a small advantage, the preferential hoarding of heavy coins[44], the preferential modern collecting of choice and therefore slightly heavier coins, and so on. The use of the tumbril or trébuchet within the letter if not the spirit of the law[45] could powerfully modify a precious-metal currency, especially where international trade was involved. The factors in the resulting pattern of weights can in practice rarely be differentiated, and attempts to analyse it in detail in terms of its discrepancy from a Gaussian distribution in any case require very large numbers of specimens[46]. Coins of the same variety but coming from different sources may differ in their weight-loss so much that combining their evidence

40. Again, it is useful to remember how the weight of a penny diverged from the pennyweight - i.e. coin names were sometimes used in the general ponderal system with a slightly different value from that of coins themselves.
41. Morrisson, 1972.
42. Miskimin, 1967.
43. Le Roy, 1971; Carcassonne and Guey, 1974; Guey and Carcassonne, 1974; Carcassonne, Dumas, and Huvelin, 1974; Müller, 1975; Guey, 1976; Carcassonne, 1977; Guey and Carcassonne, 1978 (324-326g); Müller, 1977.
44. But savings hoards may in any case consist of carefully chosen coins. Cf. the two hoards found at Neath Abbey – «two distinct hoards concealed by the same individual upon the same occasion. [One] represented his «nest-egg» or «wallet», one hundred coins carefully selected and kept back from general currency, [the other] is to be interpreted as his ready money or «purse» from which to meet day-to-day expenditure.» The coins in the purse are on average 1 gr lighter, but both histograms include a «tail». On the Neath Abbey coins see now also Müller, 1977.
45. Yvon, 1970; Mayhew, 1975.
46. Banderet, 1967. Cf. Carcassonne, Dumas, and Huvelin, 1974, p. 620.

results in utter confusion and error. It is virtually essential to consider the coins from each hoard separately : therefore one needs very large hoards.

With a smaller sample, changes in the step-interval or location of the steps of a histogram sometimes lead to startling changes of shape[47], and if one's choice is conditioned by an idea of what is normal, an element of subjective judgment cannot be excluded. A computer programme devised by S. Cope[48] lightens the task of adjusting histograms by trial and error, and can minimize the subjective element. For many purposes an approximation may, of course, be perfectly adequate, and in that case *la droite de Henry* offers a simple and useful technique of analysis. Müller has shown how a more rigorous approach can be made by using empirical moments[49].

The care with which flans were weight-adjusted varies greatly from one issue to another. One issue may produce a curve which is very compact and seemingly Gaussian; another will be flattened and irregular, with a standard deviation two or even three times as great. Successive issues of the same denomination at the same mint can differ markedly. Each issue therefore has to be studied separately[50].

Because one cannot proceed by generalization, and because so many different factors were at work modifying the weight-distribution of the coins as they circulated, the most practical approach will usually be to study two or more dateable hoards containing the same varieties of coins, one of which however was concealed twenty or more years after the other, and to compare the same issue of coins after a longer period in circulation. In this way most of the uncertainties are discounted, and, provided one is looking at a plentiful and freely-circulating base-metal or partly-token coinage, one can hope to isolate the effects of wear. Work of this kind with Frankish deniers tournois has demonstrated that flattening of the distribution-curve through wear is much less significant than the text-book diagrams contrive to imply[51]. Individual coins must admittedly receive unequal wear, but the inequalities were in practice far less in relation to the average weight-loss than Hemmy's diagram postulates. The available empirical evidence is to the effect that wear can displace the histogram along the horizontal axis to a distance of several standard deviations before the theoretically expected flattening and negative asymmetry become at all noticeable. Thus, the diagram may deceive the eye and enshrine a fallacy.

47. An illustration is given in METCALF, 1971, fig. 13, where b*i* and b*ii* represent the same data.
48. The development of the programme is described by Cope in a forthcoming paper, in *NC*, 1980. It has been used to obtain the frequency curves printed in MAYHEW, 1976, and in KING and SPAER, *NC*, 1977, p. 80-2, and KING, *NC*, 1978, p. 43. Similar examples are given e.g. in CARCASSONNE and GUEY, 1972, and NASTER, 1976.
49. GUEY, 1967, 1968a; MÜLLER, 1977.
50. ARCHIBALD, 1971.
51. METCALF, 1971.

The negative asymmetry which one frequently observes may have evolved through the selective export or melting-down of a large fraction of the currency, or it may even be an original feature of the coinage as it left the mint. But it mostly occurs, as a phenomenon, through the combination into one histogram of coins of different age. It is characteristic of currencies, and therefore of hoards, that older and correspondingly more worn coins survive *in dwindling proportions* (but not, as sketched by Nanteuil, in increasingly flattened distributions), and this automatically creates a negative asymmetry, provided that the same weight-standard was maintained throughout[52]. Claims to have demonstrated *empirically* that negative asymmetry and flattening develop in a coin population as a function of wear[53] should be received with the utmost scepticism.

The application of statistics to data relating to changes in weight-patterns needs, therefore, to be accompanied by a large measure of common sense, and by an awareness of the ways in which coins were manufactured and used[54]. A stimulating survey has recently been presented by Naster[55]. The quality of the data depends on the availability of large hoards, and the conclusions should only be generalized with caution if at all.

The statistical study of the Roman and Byzantine pound has engaged so much attention that it may be taken as a text-book example. But almost all the same theoretical considerations apply to the central weight-system of the Middle Ages – the Carolingian pound and its derivatives. Here too the numismatic evidence is of critical value. Suchodolski among others has written on the topic[56].

One should perhaps make the point that recovering detailed information about the weight-systems has not yet brought any very direct gains for monetary history. People are drawn to tackle this difficult problem, rather as they are drawn to climb a high mountain, «just because it is there». Nevertheless, as a working tool, decent habits of statistical analysis should be within the repertoire of any numismastist who handles hoards.

So much for weight-variation. Similar problems arise when we try to argue from observed metal contents to the original alloy. Accurate non-destructive measurement, particularly of trace-elements, is so difficult that the margins of error of various kinds make the statistical testing of significance hazardous. Unless differences between groups are significant at a high level of probability, one should hesitate to build much on them.

52. D.M. Metcalf, *Coins of Lucca, Valence, and Antioch*, in *Hamburger Beiträge zur Numismatik*, XXII/XXIII, 1968/69, p. 443-70, at p. 449; and see the histograms on p. 451.
53. Kosambi, 1941/2. The pursuit of asymmetry has been taken up too enthusiastically in Carcassonne, 1976, Guey, 1977, Guey and Carcassonne, 1978, and Müller, 1978.
54. Guey, 1977.
55. Naster, 1975.
56. Suchodolski, 1965; Metcalf and Miskimin, 1968; Müller, 1968; Müller, 1977.

Finally, the volume of the currency in circulation is a topic which can be studied only by the application of statistics. Attempts have been made by myself, twenty years ago, and by Eichhorn and others[57], to estimate the currency of a region by working directly from a list of the hoards and stray finds from that region; but I now think that they led into a blind alley, rather than providing a short cut. The correlation between hoarding, stray losses, and the total currency is altogether too complex and too difficult a topic to be usefully quantifiable.

The total numbers of coins in circulation are normally much smaller than the aggregate of issues from the mint, because of the export of coin in the course of foreign trade and because of wastage for a variety of other reasons. Wastage rates in modern currencies have been carefully studied, and have yielded figures of, for example, at least 0.7/0.8 per cent per annum in Britain[58]. In other words, there is reason to believe that every year in the UK as many as 70 or 80 million coins unaccountably disappear. This is not so surprising if one assumes that some of them are taken away in the pockets of 10 million tourists returning home. Moreover, if one citizen in two was so careless as to lose one coin each year, 25 million coins would be missing. So far as one can judge at present, wastage on a similar scale occurred in early times. People walked along grassy tracks rather than metalled roads; and in towns and market-places the streets were often very dirty. It is to be expected, therefore, that there would be unrecovered losses. For Sweden between 1665 and 1740 the wastage-rate has been calculated, by Thordeman in his study of the Lohe hoard, at 2% per annum. Thus it appears that fewer than one-third of the coins minted in 1670 were still in circulation compared with those of 1716[59]. Another empirical approach to the problem has been made through a study of English fourteenth-century hoards in conjunction with the documentary records of mint-output. Between ca. 1280 and ca. 1300 some 300 million coins were struck in England, but by 1325 about two-thirds of them had disappeared from circulation[60]. The historian who is interested, for example, in prices as an index of economic change, will wish to know the size of the money-supply, not the volume of mint-output. The two can be very different.

Finally, an attempt has been made to apply statistics to the stray finds of coins from archaeological sites, and, by making due allowance for the length of time individual coins remained in circulation, to obtain a more precise date bracket for the occupation of a site[61]. For sites in multiple and long-term uses, one suspects that the practical application of the method

57. METCALF, 1958; MATUSZEWSKI and WIELOWIEJSKI, 1971, 1973, 1974; LANGOUËT and GOULPEAU, 1975; COLE, 1976.
58. GLANVILLE, 1970; DOWLING, 1972; COLE, 1976.
59. THORDEMAN, 1948.
60. MAYHEW, 1976, at p. 92-5.
61. MÜLLER, 1968.

will prove limited, because there are so many factors in complex interaction, and because the coins found are such a minute proportion of those originally in use there.

To sum up: much has already been achieved through the application of statistics to numismatics. Thordeman's beautifully-written discussion of the Lohe hoard in 1948 made a considerable impact. The next two decades saw various exploratory approaches. These were then subjected to critical scrutiny, and a consensus has, I think, begun to emerge. Lines of enquiry that were unfruitful have generally been abandoned and the leading arguments have been renewed and strengthened. For the future, one may look forward to consolidation as the existing theoretical developments are applied to new material. Comparative figures, for example, for loss of weight by wear in ancient and medieval coinage should eventually establish a norm, and refine on current ideas. Nanteuil's Law states that wear will be approximately constant, in absolute terms, given equal circulation: smaller and lighter coins, in other words, lose a greater proportion of their weight through wear. Figures of 2, 2,5, 3, and 4% per decade have been proposed[62], but we need to know whether they are typical for the different sizes of coin to which they refer respectively. A long-term savings hoard, for example, or coins that had circulated in a sealed purse, might show less wear than a sum recently withdrawn from the currency.

Test-cases, where the results can be assessed on independent evidence, will be particularly helpful, and here one may point to Mattingly's discussion of the Roman Republican issues[63]. Experiments in sampling present-day coinages (such as those made at the time of decimalization in the UK[64]), and the data published regularly in mint-reports, are a valuable control, although of course the conditions of production and circulation, and the role of coinage in the money supply, have changed greatly during this century. For all earlier periods, studies based on the thirteenth and fourteenth centuries, and utilizing the mint-records, are therefore of general relevance. It remains paradoxical that monetary conditions in the ancient world, from which we have such a mass of traditional numismatic information, should be so difficult to grasp.

ACKNOWLEDGEMENTS

I am indebted to Prof. Julien Guey and to Dr. Jörg W. Müller for their friendly and extremely responsible help, given unstintingly, in the revision of successive drafts of this article. Their patient determination to get everything as correct as possible was inexhaustible, and to collaborate with them was a privilege that I appreciated deeply.

62. CRAIG, 1953, p. xvi; DOLLEY and SEABY, 1968; METCALF, 1971, p. 198-9.
63. MATTINGLY, 1977.
64. GLANVILLE, 1970; DOWLING, 1972.

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